11th Congress of the Swiss Psychological Society
August 19-20, 2009, University of Neuchâtel

Statistical Interaction between Two Continuous (Latent) Variables

Guillaume Fürst & Paolo Ghisletta

University of Geneva
Distance Learning University

Contact: guillaume.fuerst@unige.ch

Interactive effects are common and useful in various theoretical perspectives. Often tested in the context of ANOVA design, theses effects are underrepresented in regression or Structural Equation Models. Thus, this paper aims at introducing the relevance of interactive effects between continuous variables in general, and more specifically between two continuous latent variables.

Different methods to test theses interactive effects will be presented and discussed: (1) dichotomization of continuous variables and test of the interaction as in an ANOVA design (the easy, tempting “solution” that leads to serious loss of information); (2) interaction between continuous variables in a classic regression model; (3) interaction between latent continuous variables (SEM; focusing on error free variance and thus providing more statistical power, but with some noteworthy limitations).

Two empirical examples are presented (one in the field of mood-personality research, and one ion the field of creativity research); data preparation, estimation issues, graphical representations, p-value comparison and software solution are discussed.

What is a statistical interaction ?

We speak of statistical interaction when a relation between 2 variables (say X-Y) changes as a function of a third variable (say Z).

Note that the interactive effect is a multiplicative effect, the effect of the product of two IVs scores (i.e., XZ product).

A second important point to emphasis is that the interactive effect is an effect in itself, beyond and distinguished from simple effects (e.g., XZ may be significant or not, independent of the simple effects).

This kind of design is widely used in the ANOVA context – 2x2 designs, easy to test and interpret, or more complicated design.

However, it seems less frequent in a classical regression context, and very rare or emergent in latent variable models.

Technical issues might be the pretty understandable reasons accounting for that situation, and we are going to review them, and explain how to deal with them.

But before that, let's see what interaction is not to make sure the concept is well delimited.

What is not a statistical interaction?

As seen before, the interactive effect is a multiplicative effect. This is an important point to emphasis, to avoid misconception or misunderstanding – specifically, interaction is sometimes mistaken with correlation or mediation

<u>interaction ≠ correlation</u>

<u>Figure 1.</u> Figure 1a and 1b represent a kind of diagram we sometimes see to illustrate how two variables or concepts "interact".

However, in a statistical perspective that kind of diagram clearly illustrates correlation (or covariance, i.e., shared variance) and is irrelevant to represent statistical interaction.

I know this must seem pretty obvious for almost every body, but I just want to be sure to make it clear.

Figure 1a. Correlation, Venn diagram.

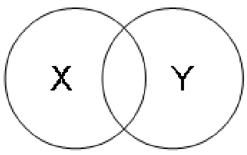
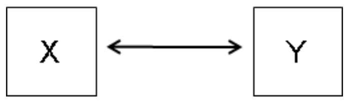


Figure 1b. Correlation, path diagram

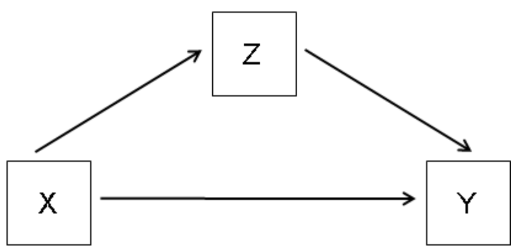


<u>Figure 2</u> represent respectively **mediation** (2a) and **moderation** (2b) path diagram (Baron & Kenny, 1986).

<u>interaction ≠ mediation</u>

Because of the complexity of multivariate data, the mediation model is sometimes mistaken with interaction. However, as we can see on the figure 2, mediation is obviously different from interaction: no XZ product, and an additional indirect X-Z-Y path.

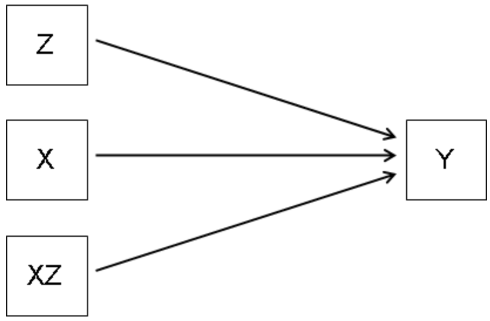
Figure 2a. Mediation path diagram



interaction ≈ moderation

Last, we can see that moderation and interaction are quite synonymous – although a moderation often implies a stronger interest in the effects of X and XZ (Barron & Kenny, 1986) –, so the figure 3 is a correct representation of interaction model (a typical classical multiple regression model).

Figure 2b. Moderation path diagram



Now that we know what the concept of interaction is and is not, let's see with concrete examples to make sure we clearly understand it (however it is tested).

Why interaction are useful

The interaction is very useful and insightful in various domain of research and subfields of psychology (clinical, social, cognitive, etc.). Here are a few concrete examples:

Example 1: *Fast driving is much more dangerous when drunk* – beyond both simple effect of driving fast (X) and driving drunk (Z), the multiplicative (XZ) effect of both is dramatic.

Example 2: *Risk of vascular disease is higher for smoker under birth control pill* – although there is no simple effect of birth control pill (X), the negative effect of smoking (Z) is even worst for women under birth control pill (XZ).

Example 3: *Medication to cure schizophrenia has a stronger positive effect for patient with high social support* – simple effects of social support (X) or medication (Z) are positive (but small), interaction between both is important (XZ).

Example 4: *Reward has a detrimental effect on creativity when people are moderately or strongly uninterested by the task, whereas it has a positive effect for the strongly interested people* – simple effect or reward is negative (X), simple effect of interested (Z) is positive, and XZ shows that the mean effect of reward is reversed for interested people.

Several interesting points can be drawn from these examples:

First, as noted before, we see that many different pattern of results are possible, (the interaction XZ might be significant, whether the simple effect of X and Z are positive, negative, or non-significant).

Second, in practice, the distinction between moderation and mediation might not be as easy as it looked on abstract graphs of previous slide (figure 2a and 2b)

For example, alcohol and fast driving interact, but a mediation effect might also be likely; alcohol may provoke fast driving.
In such cases, theoretical reflexion is extremely important, to decide what to test and how to appropriately test it. – Moderation and mediation can be integrated, but we will not get into it here (see Edwards & Lambert, 2007).

Last, note that most of these variables can be conceptualized as continuous, allowing so a finer understanding of these concepts and their relationships.
For that reason, continuous variable, probably more informative, should be preferred whenever it is possible – moreover, dummy variables such as “medicated/not medicated”, “fast/not fast”, “motivated/unmotivated”, etc., are quite a poor way to conceptualize things. So, don’t force continuous variables to be categorical to apply models such as ANOVA!

Now that we are clear with the definition and usefulness of interaction between continuous variables, let’s see how it works technically.

How to test interaction between continuous variables

In the next parts of this presentation, we will compare 3 ways to test interaction between continuous variable:

(1) based on dichotomized manifest variables – a poor way, although quite frequently used;
(2) based on continuous manifest variables – a good correct way, no so difficult that it could sometimes seems;
(3) based on continuous latent variables – the newer, promising method.

About (1), there is a great consensus in the statistical literature to say that throwing away information about your variable is never a good idea, and throwing away observations (e.g., extreme group approach) neither (e.g. MacCallum et al., 2002; Preacher et al., 2005).

The reasons are related to:
- loss of information about individual differences;
- loss of power (because of sample size reduction);
- bias in standard error and r^2 estimations (both over and under estimate);
- problems related to regression to the mean (extreme score are less reliable);
- problems related to compare findings across studies (splitting point is data dependant and changes across studies);

For these reasons, we will not discuss in details nor encourage the techniques using dichotomization of continuous variables (e.g., median split, extreme group approach) – see references if you want to know more about this.

Rather, we will focus our interest on methods (2) and (3). However, to prove our point, results coming from this approach will be compared with results coming from the two other more accurate approaches. But before this results comparison, let’s see in more details how we proceed to with methods that does not throw information of our continuous variables.

Multiple Linear Regression (MLR) strategy – interaction between two manifest continuous variables

Important points (e.g., Cohen, Cohen, West, & Aiken, 2003):

(a) Main effects should be included in the model (even if non significant), in order to avoid confusion between simple effect and interaction;

(b) The data must be multivariate normal (no outlier, normally distributed residuals, and homogeneity of variance);

(c) The predictors should be centred (i.e. subtract the mean to all scores) before you create the interaction term. There are two reasons for doing such a thing – which might seems obscure at first glance.

(1) The first and very important reason is to avoid collinearity issues (here, specifically, this means avoid strong correlation between the interaction term and the variables from which it is calculated).

As it can be seen in the left side of table 1, when the predictors are not centred the XZ interaction term is likely to correlate with X and Z (high scorers on X, in grey, are also high scorers on XZ; and the same is true for high scorers on Y, in red).

Centring the predictors (right side of table 1) leads to a different situation, where XZ is less likely to correlate with X and Z (high scorers on X, in grey, have high or low score on XZ; and the same is true for high scorers on Y, in red).

Table 1. Fictive database with centred and non centred predictors

	non centered variables			centred variables (Xi – Mx)		
	X	Z	XZ	X_c	Z_c	XZ_c
Person 1	10	3	30	4.5	-8.5	-38.25
Person 2	10	20	200	4.5	8.5	38.25
Person 3	1	3	3	-4.5	-8.5	38.25
Person 4	1	20	20	-4.5	8.5	-38.25
Mean	5.5	11.5		0	0	
r with XZ	0.65	0.59		0	0	

(2) The second reason for centring the predictor is to facilitate interpretation. To illustrate this, let's consider the equation of a basic model with 2 simple effects and interaction:

$$Y = \beta_0 + \beta_1(X\_c) + \beta_2(Z\_c) + \beta_3(X\_c)(Z\_c) + e$$

Where

β_0 is the intercept: score on Y for people who have a mean score (0) on X, Z and XZ;
β_1 is the coefficient of the simple effect of X ;
β_2 is the coefficient of the simple effect of Z ;
β_3 is the interactive effect, the multiplicative effect of XZ;

Then if, for the sake of the example, we **use the following fictitious value**:

$$\beta_0 = 0 \qquad \beta_1 = 1 \qquad \beta_2 = 1 \qquad \beta_3 = 1$$

We can see that it is **very easy to write equation** (seriously):

Example 1. Equation for high scorer on X (say 10) with mean score on Z (0):

$$Y_{pred} = 0 + \underline{1(10)} + 1(0) + 1(10)(0) = 10$$

(which correspond to the simple effect of X)

Example 2. Equation for high scorer on X (say 10) and Z (say 10):

$$Y_{pred} = 0 + 1(10) + 1(10) + \underline{1(10)(10)} = 120$$

(which illustrate well the multiplicative effect of XZ)

Although they might seem pointless now, theses considerations will reveal all their relevance when we look how to construct the interaction graph…

Structural Equation Modelling (SEM) strategy – interaction between two latent continuous variables

In the context of SEM, the basic idea stays the same – two X and Z predictors and an XZ interactions term which correspond to the multiplicative effect.

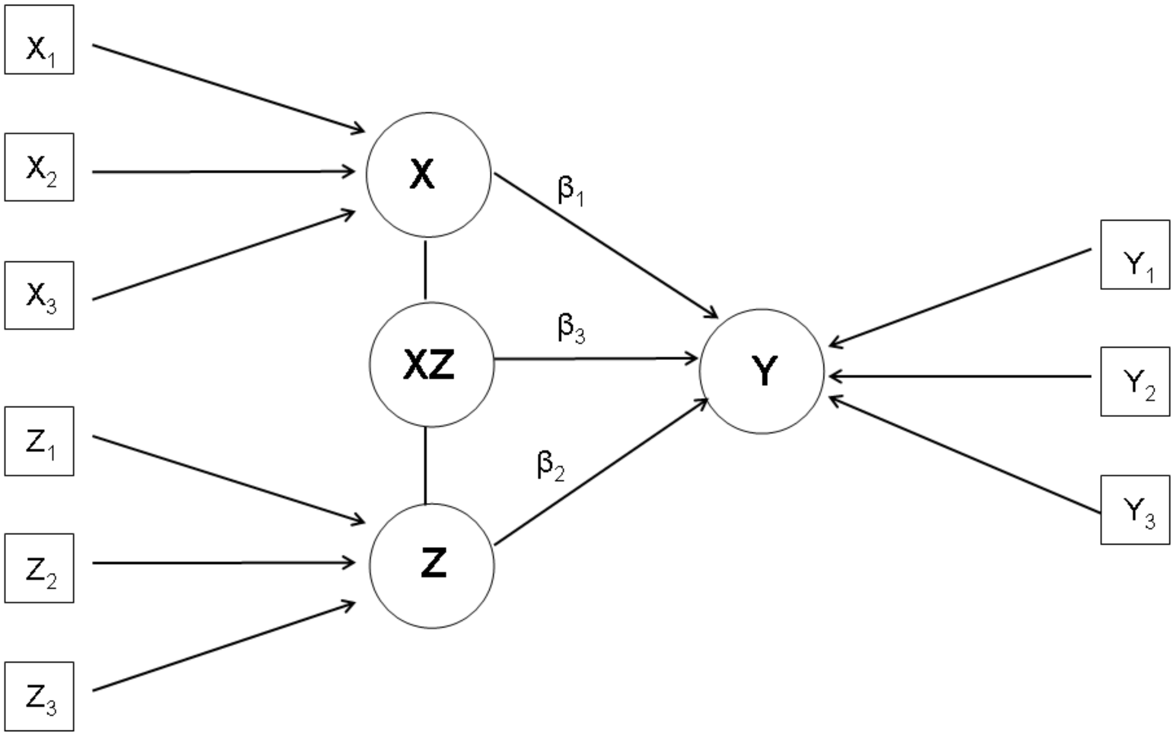
The main change here is that variables are no more “simple” manifest variables (for example, one unique mean score to a questionnaire), they are factor, or latent variables, extracted for multiple indicators (for example, shared variance between every items of a questionnaire) – see figure 3 below.

The main (and huge) advantage of such a model is to clearly distinguish the true variance (variance of factor) and the “error” variance (residual variance).

As in RLM model, the estimation method is maximum likelihood – other methods exist, still in development, (e.g., Marsh et al., 2004; Schumacker, & Marcoulides, 1998);

However, a limitation might be that this solution is directly implemented on few software only (e.g., Mplus) – see example below. However, intermediate solution or compromise exists (see practical suggestions in the conclusion section and/or Schumacker, & Marcoulides, 1998, for example).

Figure 3. Example of interaction model with latent variable.



Example of Mplus script corresponding to fig. 3:

Analysis' type	`ANALYSIS:`
	`TYPE = RANDOM;`
	`ALGORITHM = INTEGRATION;`

Factors' definition	`MODEL:`
	`X BY x1 x2 x3;`
	`Z BY z1 z2 z3;`
	`Y BY y1 y2 y3;`

Interaction term	`XZ | X XWITH Z;`
Regression paths	`Y ON X Z XZ;`

Empirical examples

We present 2 examples were, the variables used as described below.

Example A.

X = inspiration-like processes, 6 items (1-5 likert scale), e.g., « have crazy ideas », « explore many different possibilites »; Z = deep processing and work immersion, 6 items (1-5 likert scale), e.g. « evaluate ideas », « verify, consider details ».
Y = Creative activity, 3 items: time spent per week on creative or artistic activities (e.g., music, writing, painting), seriousness of the activity and achievement (e.g., social recognition, prize won).

Example B.

X = extraversion (E), 9 items (1-5 likert scale), e.g., « enthusiastic », « seek social contact »; Z = neuroticism (N), 9 items (1-5 likert scale), e.g. « worry easily », « experience mood swings ».
Y = unpleasant mood, 6 items (1-5 likert scale), e.g. « feel unhappy », « feel bad ».

All variables used in this example are scores from self-report. Items have been either (1) summed and dichotomized; (2) just summed; or (3) use to estimate factor scores.

Table 2. Results of the 2 examples within the 3 methods.

(A) Creativity-Process Data

(1A) Dummy Manifest Variables

	Estimate	Std. Error	p-value
INSP	0.42	0.08	> 0.0001
WORK	-0.01	0.08	0.89
IxW	**0.13**	**0.08**	**0.11**
R^2	**0.21**		

(2A) Continuous Manifest Variable

	Estimate	Std. Error	p-value
INSP	0.49	0.08	> 0.0001
WORK	0.02	0.08	0.80
IxW	**0.20**	**0.08**	**0.01**
R^2	**0.34**		

(3A) Continuous Latent Variables

	Estimate	Std. Error	p-value
INSP	0.8	0.26	0.003
WORK	0.25	0.17	0.14
IxW	**1.6**	**0.55**	**0.002**
R^2	**0.42**		

(B) Mood-Personality Data

(1B) Dummy Manifest Variables

	Estimate	Std. Error	p-value
E	-0.36	0.10	0.0003
N	0.22	0.10	0.025
ExN	**-0.16**	**0.10**	**0.107**
R^2	**0.19**		

(2B) Continuous Manifest Variable

	Estimate	Std. Error	p-value
E	-0.41	0.09	> 0.0001
N	0.31	0.09	0.0007
ExN	**-0.16**	**0.09**	**0.0665**
R^2	**0.25**		

(3B) Continuous Latent Variables

	Estimate	Std. Error	p-value
E	-0.48	0.11	> 0.0001
N	0.37	0.12	0.002
ExN	**0.44**	**0.20**	**0.025**
R^2	**0.43**		

Dichotomization leads to lower p-value and R^2 than both RLM and SEM method.
SEM seems the overall most accurate and powerful method.

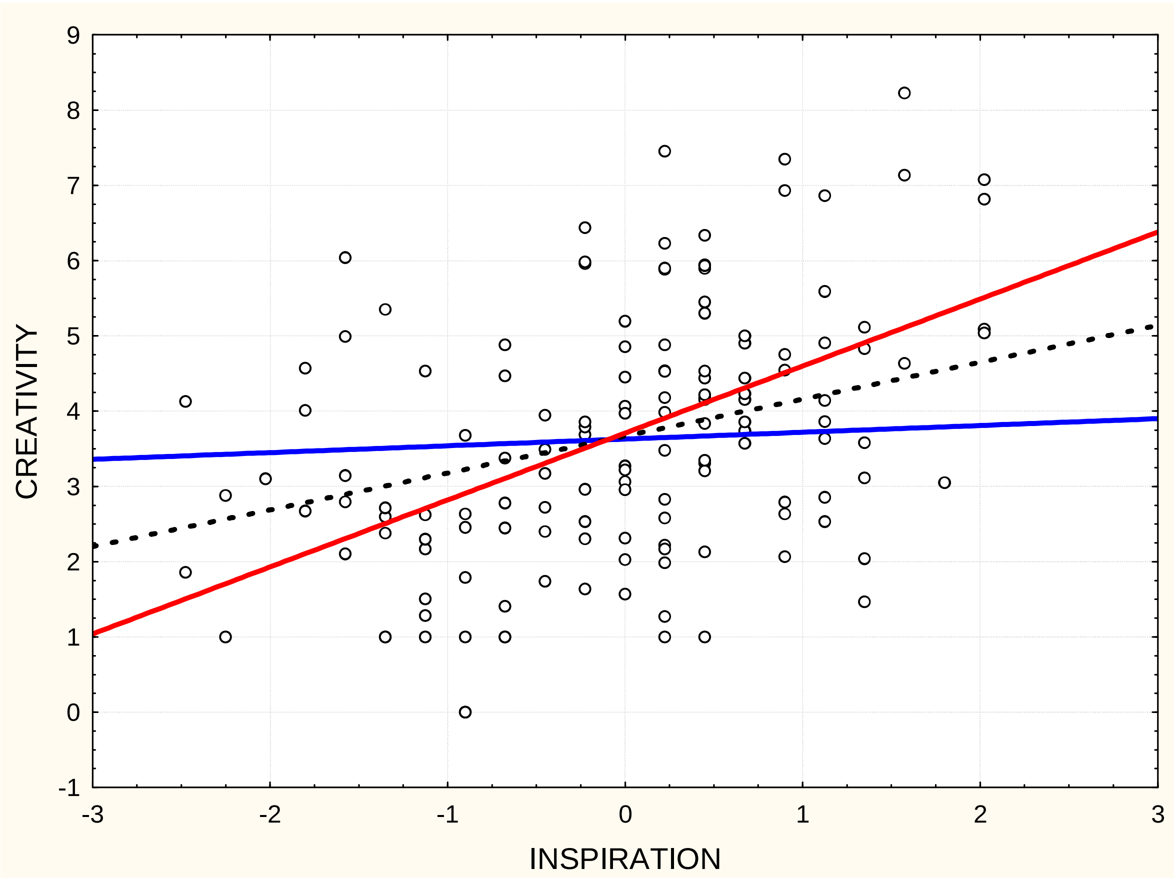
Graphs & Interpretation

Example A. Creativity, work and inspiration

For high "work" scorers (**+2 S.D. above the mean**),
Creativity = 3.67 + 0.49*(x) + 0.02*(**+2**) + 0.20*(x)*(**+2**)

For mean "work" scorers (**0 S.D.**),
Creativity = 3.67 + 0.49*(x) + 0.02*(**0**) + 0.20*(x)*(**0**)
= simple effect of inspiration

For low "work" scorers (**-2 S.D. above the mean**),
Creativity = 3.67 + 0.49*(x) + 0.02*(**-2**) + 0.20*(x)*(**-2**)



Interpretation:

X: Inspiration is positively related to creativity (no huge breakthrough here…)

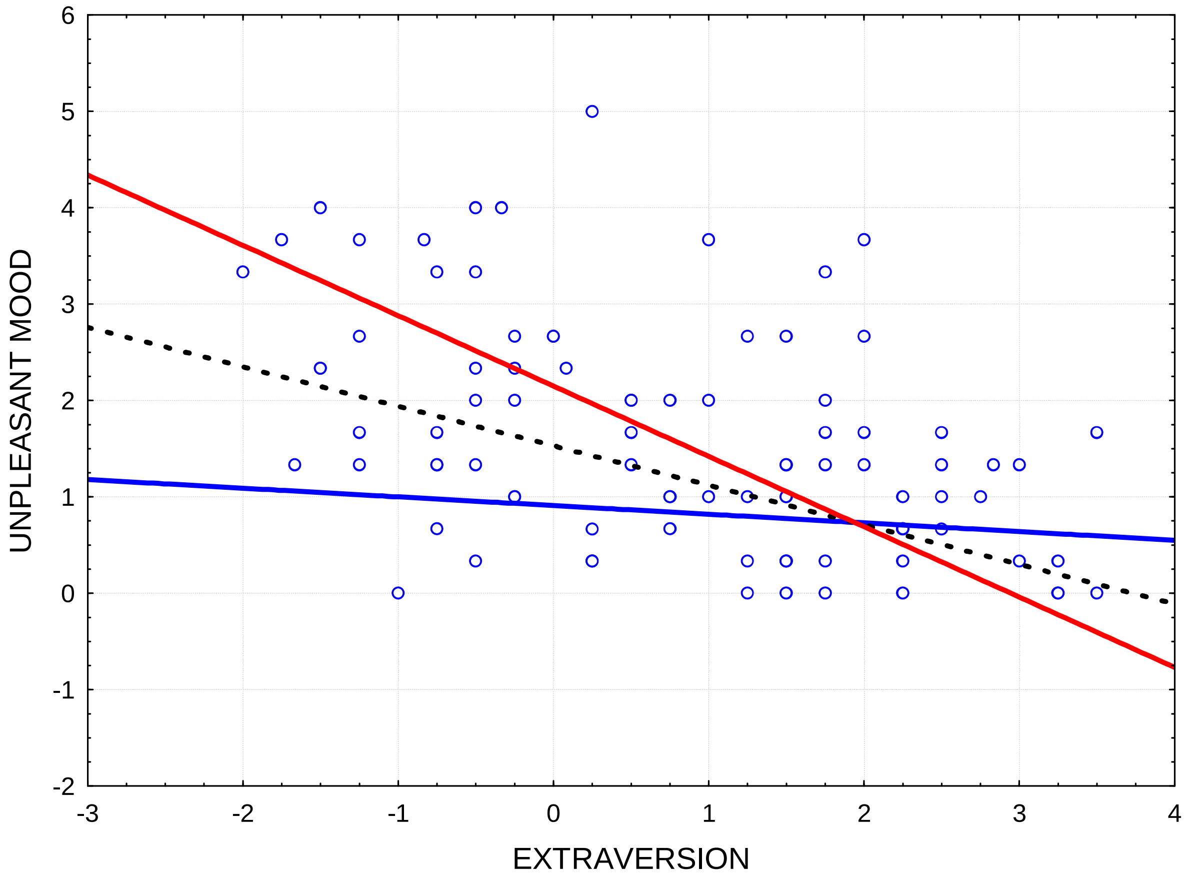
XZ: The positive effect of inspiration on creativity is stronger for people who work hard (no surprise either!). Conversely, uninspired people, even if working hard, are not likely to be creative (I know this is cruel, but the data tell so).

Example2. Neuroticism, mood and extraversion

For high "Neuroticism" scorers (**+2 S.D. above the mean**),
Unpleasant mood = 1.53 + (-0.41)*(x) + (0.31)*(**+2**) + (-0.16)*(x)*(**+2**)

For mean " Neuroticism " scorers (**0 S.D.**),
Unpleasant mood = 1.53 + (-0.41)*(x) + (0.31)*(**0**) + (-0.16)*(x)*(**0**)
= simple effect of extraversion

For low "Neuroticism" scorers (**-2 S.D. above the mean**),
Unpleasant mood = 1.53 + (-0.41)*(x) + (0.31)*(**-2**) + (-0.16)*(x)*(**-2**)



Interpretation:

X: Extraversion is negatively related to unpleasant mood.

Z: Neuroticism is positively related to unpleasant mood (risk factor for depression)

XZ: Non-neurotics benefit less of the positive implications of extraversion. Or, conversely, the detrimental effect of neuroticism is stronger for introverts.

Conclusion & recommendations

General points to remember:

Interactions between continuous variables have great conceptual meaning and relevance.

Interactions between continuous variables are quite easy to test and to represent graphically.

The (abusive) practice of dichotomization should be avoided in most cases (unless motivated by taxometric or latent class-analysis), because it generally leads to serious loss of accuracy (smaller p-value, lower R^2).

Structural Equation Models (SEM) offer a great framework to include interaction in a broader context (factorial models, regression models) and to test them more accurately – SEM should be preferred when multiple indicators are hands.

And some pragmatic advice:

Look for interactions, make sure that is really what you want to test (not a mediation, or moderated mediation, etc.)

Plan carefully your study and prefer continuous variables and keep them continuous – even if the distributions are non-normal or the relation with the dependant variable is non-linear. A transformation (e.g., log(x), x^2) is a much better way to solve these specific issues.

At least, test your interaction with a Multiple Linear Regression (center your predictors for an easier interpretation and graph)

At best test your interaction with a Structural Equation Model (SEM), using multiple indicators for each variable (centring predictor is often not necessarily required in that case)

A pragmatic compromise between the two solutions may consist in first estimate factor scores, center them, calculate the XZ product between them, and then follow the MLR procedure.

In any case, include the simple (main) effects in the model.

And beware of multicollinearity (i.e., check correlation between predictors), use theoretical justification and/or stepwise techniques if you encounter that kind of problem.

References

Interaction and mediation in the RLM context

Baron, R. M., & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic and statistical considerations. Journal of Personality and Social Psychology, 51, 1173-1182.

Cohen, J., Cohen, P., West, S.G. & Aiken, L.S. (2003) Applied Multiple Regression/Correlation Analysis for the Behavorial Sciences. Lawrence Elbaum Associates, London. [see chapter 7]

Dichotomization and Extreme Goupe Approach

MacCallum et al. (2002), On the practice of dichotomization of quantitative variables, Psychological Methods, 7 (1), 19-40.

Meehl, P.E. (1992). Factors and taxa, traits and types, differences of degree and differences in kind. Journal of Personality, 60, 117-174.

Preacher, K. J., Rucker, D. D., MacCallum, R. C., & Nicewander, W. A. (2005). Use of the extreme groups approach: A critical reexamination and new recommendations. Psychological Methods, 10, 178-192.

SEM introduction

Hoyle, R. H. (Ed.). (1995). Structural equation modeling. Thousands Oaks, C.A.: Sage Publications.

Byrne, B. (2001). Structural equation modeling with AMOS: Basic concepts, applications, and programming. Mahwah, NJ: Lawrence Erlbaum

Going further with SEM

Edwards J.R., & Lambert L.S. (2007). Methods for Integrating Moderation and Mediation: A General Analytical Framework Using Moderated Path Analysis. Psychological methods 12(1), 1-22

Schumacker, R. E., & Marcoulides, G. A. (Eds.). (1998). Interaction and nonlinear effects in structural equation modeling. Mahwah,NJ: Erlbaum.

Marsh, H.W., Wen Z., & Hau K.-T. (2004). Structural Equation Models of Latent Interactions: Evaluation of Alternative Estimation Strategies and Indicator Construction. Psychological Methods, 9 (3), 275–300